Inflation Uncertainty: ETF Risk Model, Structural Change, and Breakeven Inflation

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Abstract

We present a predictive risk model for a popular Treasury Inflation Protected Securities (TIPS) ETF. The model is based on ARMA-GARCH and a heavy-tailed volatility model. Its assessment based on standard VaR, CVaR, and Kolmogorov-Smirnov statistics is very encouraging. We also study structural breaks in the CPI-based inflation rate. The structural breaks correlate with key inflation related external events. Inflation expectations are expressed through the breakeven inflation (BEI) rate. We infer distinctive market motivations of TIPS investors based on BEI expectations across TIPS maturities.

Keywords: Inflation, Breakeven Inflation Rate, TIPS, Risk Model

1. Introduction

1.1. Treasury Inflation-Protected Securities

TIPS are fixed income securities issued by the U.S. Treasury. As with comparable US Treasury bonds (referred to here as nominal bonds), they pay interest twice a year at a fixed rate. The principal of a TIPS bond is tied to the value of the Consumer Price Index (CPI). As the inflation rate changes—as measured by the CPI—the par value of the TIPS is adjusted by the same rate. This feature allows protection from inflation to TIPS purchasers. The payoff at maturity is the inflation adjusted par value or the original par value, whichever is larger. Due to their inflation insurance property, TIPS almost always carry lower interest rates than nominal bonds.

1.2. Breakeven Inflation Rate (BEI)

The breakeven inflation rate compares nominal fixed income securities with inflation-indexed securities. The breakeven inflation rate is calculated as the difference between the yield of nominal treasury securities and TIPS of the same maturity. It is also the average annual inflation rate required for a treasury inflation indexed security (TIPS) to have the same return as an equal maturity treasury bond. The breakeven inflation rate is a key component of the inflation risk/uncertainty premium. This premium reflects an expectation of the future inflation rate and includes the market value of inflation risk associated with uncertainty in the future inflation rate. The value of the risk premium is perceived differently across TIPS investors of different maturities as we will discuss later.

This BEI analysis of this paper contains two sections. The first section observes the term structure of the 5-year, 20-year, and 30-year BEI rates. The second section calculates a realized inflation uncertainty premium using 5-year, 7-year, and 10-year BEI rates. Specifically, we are interested in monthly resolution data. Ref. [3, 4, 6] provide the 7-year, 20-year, and 30-year monthly BEI rates with no missing points. Ref. [5, 7] provide daily resolution BEI data for 5Y and 10Y TIPS respectively and was missing about 10% of its data in our time range. The daily data was converted to monthly data using only the first business day of every month. Missing data were computed by averaging adjacent daily values.

1.3. Inflation Rate

The inflation rate is calculated using the non-seasonally adjusted CPI [2] at monthly resolution. The annualized inflation rate at month n is

$$Inf_n = \frac{CPI_n}{CPI_{n-12}} \tag{1}$$

This monthly inflation rate is used for all further comparisons. Fig. 1 compares the BEI from various maturities to changes in the annual inflation rate.

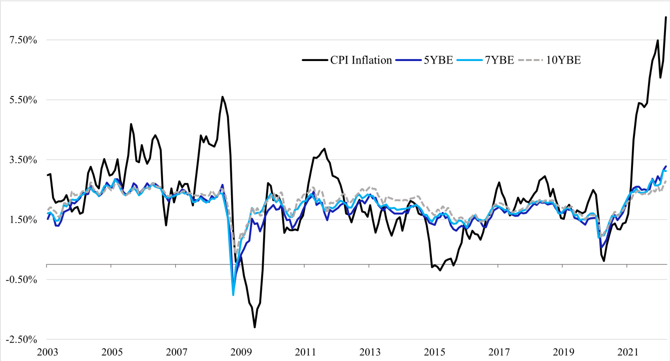

Figure 1: 5Y, 7Y, and 10Y BEI Rates and monthly inflation rate (Feb 2003 - Apr 2022)

2. Literature Summary

Many papers discuss TIPS and BEI as an expectation of the inflation rate. A common conclusion is that the BEI contains both an expectation of future inflation and a demand premium know as the inflation risk premium. This idea is discussed in [9].

Ref. [12] annualizes the inflation rate over the life of TIPS bonds, a method is used in section 3 of this paper. It states that breakeven inflation rates are reasonable approximations of CPI based inflation and revolves around the accuracy of BEI based predictions. It also concludes that longer term TIPS investors tend to overestimate future inflation compared to shorter term TIPS investors who underestimate future inflation.

Risk models are based on a combination of ARMA – GARCH [13] methods to remove trends and a heavy tail model to capture non-Gaussian extreme events. This leads to a better estimation of variance. We are not aware of previous studies of risk models applied to TIPS ETFs.

3. Risk Model with GA 1.2

A proprietary risk model (GA 1.2), based on the above ideas, is used to assess daily portfolio risk of holdings in BlackRock's iShares TIPS ETF. We introduce three metrics for the assessment of the risk model. The first is the Kupiec test, which assesses the confidence of Value at Risk (VaR). The second assessment is is a statistical t-test which measures the confidence value of Conditional Value at Risk (CVaR) on the hypothesis that the returns on VaR excedances are t-distributed. The third assessment method is the Kolmogorov-Smirnov (KS) statistic which measures return accuracy at all quantiles. These tests are standard across literature evaluating risk models.

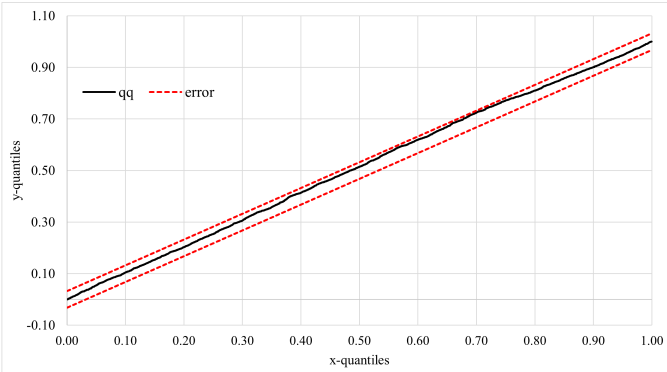

Figure 2: QQ plot of modeled ETF returns against sample returns

Fig. 2 is a quantile-quantile plot that compares historical returns by quantile on the x-axis to the model predicted returns by quantile on the y-axis. The error limits are the 5% significance point for a KS test for a sample n = 50. There are no exceedances of the error limits. The Kolmogorov-Smirnov (KS) statistic of the distribution has a p value of 0.1929. The VaR and CVaR models are also very encouraging. Fig. 3 plots iShares TIPS ETF returns across time along with our 95% VaR and CVaR values.

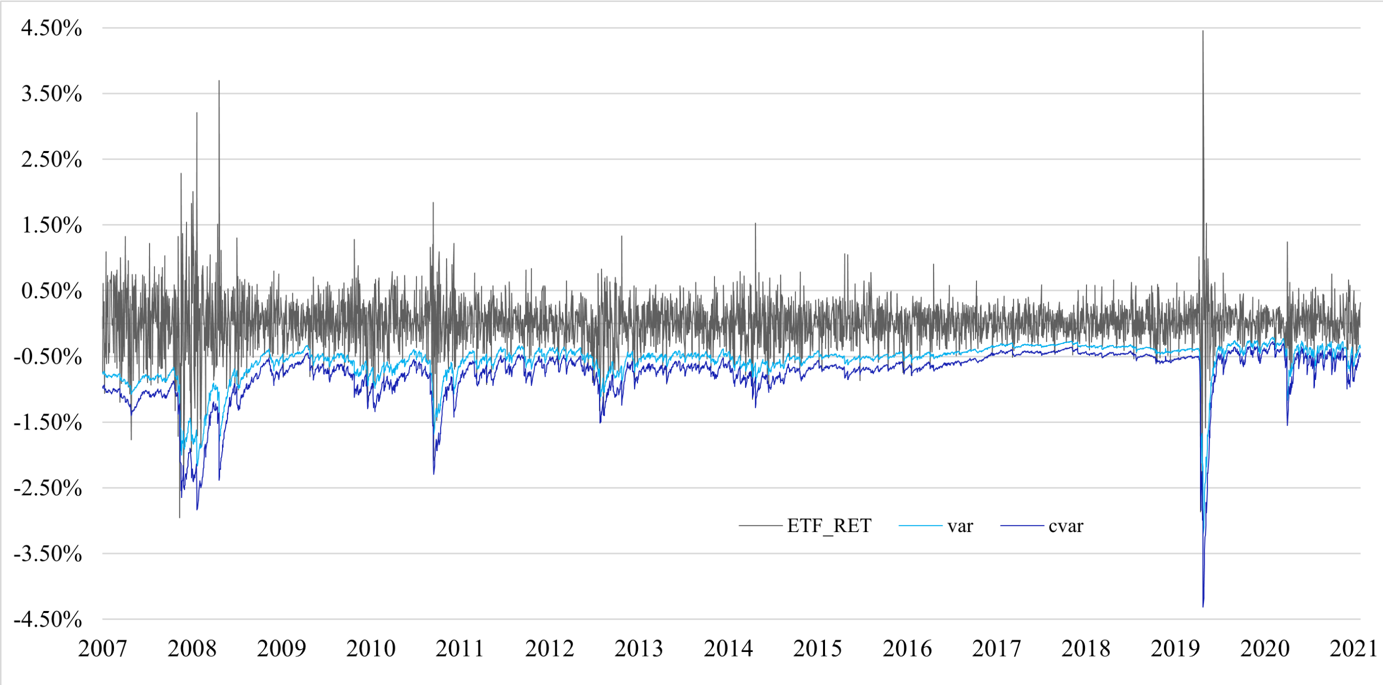

Figure 3: TIPS ETF Returns with 95% VaR and 95% CVaR

The Kupiec test [14] delivers a p-value for the VaR model at each quantile. A similar process takes place for CVaR with a t-test for significance. Table 1 shows the p-values for VaR and CVaR at the 95th an 99th percentile.

Table 1: p-values for VaR and CVaR statistics

Quantile	VaR	CVaR
0.95	0.305	0.242
0.99	0.942	0.135

4. Structural Breaks

The Chow F-statistic [15, 16] is a statistical indicator of a structural change point. We analyze a time series of F-statistics based on the CPI inflation rate. Relative peaks in the curve are viewed as evidence of change points, following the methodology of [15] . We relate these change points to concurrent political and economic events.

We apply ARMA(1,1) to the CPI-inflation time series to generate the residuals, and then calculate the Chow F statistics on this series. The F statistics are as follows.

$$F = \frac{RSS - ESS}{ESS(n-2k)} * 1/k \ . \quad (2)$$

ESS is defined as the total error sum of squares from estimating the *2k* parameters of the model before and after each candidate change point. RSS is defined as the restricted residual sum of squares calculated from using the same model to estimate$k$ parameters using the entire time series. The F statistics carries k and *n-2k* degrees of freedom.

Fig. 4 displays the Chow F-statistics of the CPI Inflation series. We identify four significant change points from the Chow F-statistics. We then observe that these large spikes in the F-statistic curve correlate with key economic events.

The first region is the 2008 recession highlighted in the first gray section of the plot with dates Dec 2007 – Jun 2009. The period contains both the economic bubble and its bursting. We see a spike in the F-statistic series during both the bubble and its burst. The second gray region highlights the COVID-19 stock market drawdown from Feb

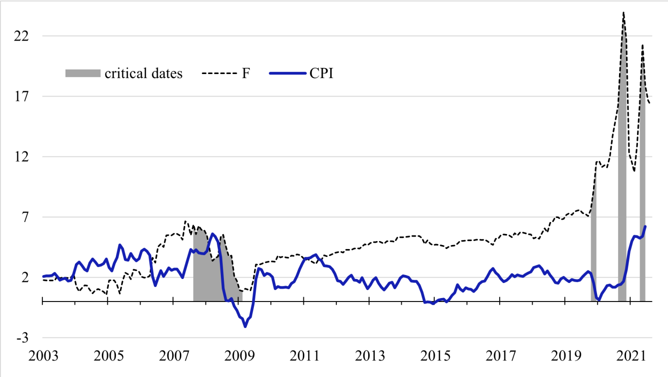

Figure 4: F-statistics for structural change in CPI-based inflation (May 2003 – Dec 2021)

2020 – Apr 2020. Both recessions display evidence of a change point in the F-statistic curve. A recessionary event is likely to cause significant changes in the inflation time series and also be material to TIPS investors.

The third gray region is Jan 2021 – Mar 2021. This period historically is marked by a post COVID-19 economic recovery and the inauguration of a new president. The 2020 presidential election was historically contentious and marked a shift in the political landscape focusing on the economic policies of a new president, not yet clearly defined. These phenomena contribute to inflation uncertainty during this region and are accompanied by record high BEI rates. The fourth gray region is marked from Aug 2021 – Oct 2021. This region was marked with another increase in inflation while consumers had expected inflation to slow down. This increase of inflation in Sept 2021 accompanies another spike in the Chow F statistics suggesting a change point. This change point is also accompanied by increases in BEI rates.

Out of the four historical regions, two regions were inflationary and two regions were deflationary. Structural change points in recessionary periods accompany negative changes in CPI-inflation and BEI rates from all maturities. Change points during inflationary periods such as in 2021 accompany large positive percent changes in CPI-inflation and BEI rates.

5. BEI Expectations

We examine BEI rates across various maturity TIPS to observe inflation expectations. We observe changes in BEI term structure when examining 5Y, 20Y, and 30Y monthly BEI rates.

Fig. 5 shows that the 20-year and 30-year BEI rates are larger than the 5-year BEI rate for most of the observed period. This appears reasonable since a longer prediction of inflation carries a larger inflation risk premium. However, starting 2021, the term structure of the observed BEI rates is reversed. The 5-year BEI rate or the 5-year inflation expectation is larger than both the 20-year and 30-year expectation.

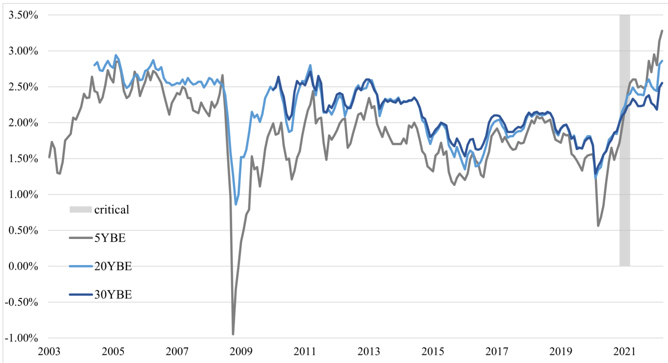

Figure 5: 5Y, 20Y, and 30Y BEI rates

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6. BEI Expectations vs Realized Inflation Rate

We determine the predictive power of these BEI rate inflation expectations by relating them to the average annual CPI-based inflation rate through the life of the bond. We calculate the average annual inflation rate for n-years using the compounded annual growth rate (CAGR) formula as follows.

$$CAGR = \left(\frac{V_T}{V_0}\right)^{\frac{1}{T}} - 1 \tag{3}$$

The difference between the n-year BEI rate minus the n-year CAGR inflation rate. Equation 4 represents the spread between predictions and realized inflation. We interpret this spread as the realized premium paid for inflation protection across time and TIPS maturities.

$$P_t = BEI_t - \left(\frac{CPI_{t+n}}{CPI_t}\right)^{\frac{1}{n}} - 1 \tag{4}$$

We can interpret this premium P in the context of three approximate economic cases.

$P = 0$: The BEI rate correctly estimates the true average inflation rate over the life of the TIPS. An investment in TIPS yields a return equal to a nominal treasury. Inflation risk is correctly valued with a zero cost of uncertainty.

$P \leq 0$: The BEI rate underestimates the true average inflation rate over the life of the TIPS. An investment in TIPS yields a greater return than a nominal treasury. Inflation risk is undervalued and TIPS investors are receiving a premium.

$P \geq 0$: The BEI rate overestimates the true average inflation rate over the life of the TIPS. An investment in TIPS yields a smaller return than a nominal treasury. Inflation risk is undervalued and TIPS investors are paying a premium.

Fig. 6 shows the inflation risk premium paid as a time series. During recessionary regions, inflation risk is undervalued and TIPS investors receive a premium represented by negative values in the graph. In other regions, inflation risk is overvalued with TIPS investors paying a premium for inflation protection. The highlighted regions indicate key inflation-related events where inflation risk was heavily undervalued. The first region is from March 2003 – July 2003. In this region, the Iraq War is announced. Inflation risk premium is undervalued during this period. The second gray

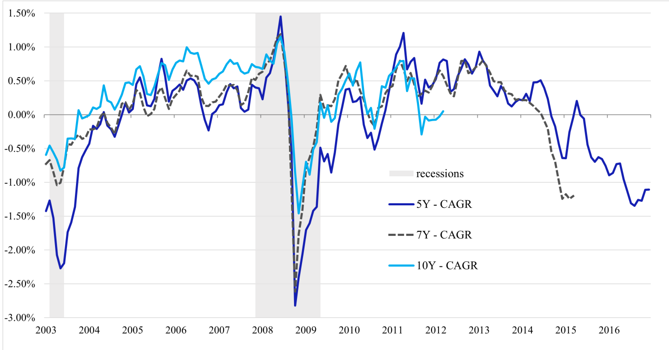

Figure 6: Inflation Risk Premium measured for 5Y, 7Y, 10Y TIPS

region is from Dec 2007 – June 2009. This region contains the peak effect of the housing bubble preceding the 2008 economic crash. Before the crash, inflation risk carries its largest observed premium. The succeeding recession period carries the smallest observed premium and is also a noted change point in Fig. 4. We observe that recessionary events with historical context accompany regions of undervalued inflation risk. Table 2 summarizes the statistics of this metric for the three different TIPS maturities.

Table 2: Inflation Risk Premium Summary Statistics (%)

	5Y	7Y	10Y
Mean	-0.087	0.15	0.29
Median	0.14	0.27	0.43
SD	0.78	0.55	0.52
n	171	147	111

Inflation risk premium increases over longer maturities. Longer term TIPS investors tend to overestimate inflation risk while shorter term TIPS investors slightly underestimate inflation risk. This result agrees with observations from [12]. We hypothesize TIPS investors motivations into two broad categories.

Long term TIPS investors' interest is in wealth appreciation and portfolio optimization. Speculating/hedging during recessionary periods is not of extreme interest. Further, long term TIPS purchases can be automated as part of meeting regulatory and diversification requirements for pension funds, retirement portfolios, etc. These reasons create a demand for inflation protection which reflects in higher premiums.

Shorter term TIPS investors' interest is in market hedging, speculation, and the risk of future inflation. TIPS demand and hence premiums are more sensitive to recessions and inflation scares within recessionary and other inflation related events. This increased sensitivity to inflation uncertainty is reflected in the volatility of the 5Y premium. Both the largest and smallest observed premium come from 5Y TIPS investors.

These two reactions support the inference of differing market motivations in the TIPS market. Further, the standard deviation of this metric decreases across maturities, which is further evidence of a decreasing speculative sentiment as maturities get larger.

7. Conclusions

A risk modeling code (GA 1.2) [1] is successful in modeling future probability distributions of portfolio returns and risk based on a popular TIPS ETF. The model is assessed using confidence values for VaR/ CVaR, and a KS statistics for the entire distribution. All three statistics have significant p-values.

We find that structural breaks in the CPI-based inflation time series are significant in two recessionary periods and several recent inflationary periods. Change points associated with inflationary periods accompany positive changes in CPI-inflation and BEI rates. Change points associated with deflationary periods accompany negative percent changes in CPI-inflation and BEI rates. Most change points are associated with notable historical and economic events likely to affect TIPS investors.

Our study captures a general sense of inflation risk premium by comparing the BEI rate to a compounded annual inflation rate through the life of the TIPS. The gap between the two is interpreted as an inflation uncertainty premium and compared relatively across investors of differing TIPS maturities. We find that TIPS investors of longer maturity TIPS tend to overestimate inflation on average and carry a small speculative sentiment. TIPS investors of shorter maturities tend to underestimate inflation on average but carry a larger speculative sentiment.

From this, we hypothesize two different motivations of short term and long term TIPS investors. Shorter term investors are concerned more with market hedging and protecting short term assets against inflation. Longer term TIPS investors are concerned with appreciating wealth over longer periods and are less interested in inflation speculation.

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